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smoothly as possible on the 6th, 7th, 8th, and 9th, and up to 11½ P.M. on the latter day (9th), suddenly a great downward jerk in the curve, forming a gap as far as 3½ A.M. on the 10th. Then comparative tranquillity till 11 A.M., and then (corresponding to the re-absorption of the spots) a furious and convulsive state of disturbance extending over the 11th and the greater part of the 12th. I wonder whether anything was shot out of those holes on that occasion! and, if so, what is going on in the inside of the Sun?"

Note on a special point in the determination of the Elements of the Moon's Orbit from Meridional Observations of the Moon. By George Biddell Airy, Astronomer Royal.

The magnitude of the Moon's apparent disk produces a difficulty in the reduction of observations of the Moon which does not occur in the case of the Sun or any of the Planets. In general, the two opposite limbs of the Moon (either those observed in Right Ascension or those observed in North Polar Distance) cannot be fully illuminated, and, in the great majority of instances, they cannot even be in a state approximating to that of full illumination; no attempt can be made to observe the centre; and therefore the only way of determining the apparent place of the centre is, to observe the fully illuminated limb, and to apply to the result a numerical correction representing the computed value of apparent semidiameter at the time of observation. This apparent semidiameter must be computed from a mean semidiameter inferred from observations of both limbs, when both limbs are so near to the state of full illumination, that the small correction depending on the deficiency of illumination of one limb can be calculated with great accuracy. Very few observatories furnish observations adapted to this purpose; and in consequence there will always be a little uncertainty about the value of the apparent semidiameter.

In correcting the observed Right Ascension of a limb, there will therefore always be a small uncertainty on the inferred Right Ascension of the centre, which has one sign before Full Moon and the opposite sign after Full Moon. It is well known that this introduces uncertainty with regard to the coefficients of the two important equations designated the "Variation" and the "Parallactic Equation." These effects, however, are sufficiently well known, and I shall not delay further on them.

The point to which I invite the attention of the Society is one which I believe has never been noticed, namely, the effect of the uncertainty of correction for semidiameter on the inferred results for North Polar Distance of the Moon's centre. I believe that it is not generally known to the investigators of Lunar Theory and

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the constructors of Lunar Tables (it was not known to me till I had been a practical astronomer of some years' standing), that near the summer solstice, or (roughly) from the beginning of May to the end of July, only the North Limb of the Moon can be observed; and near the winter solstice, or from the beginning of November to the end of January, only the South Limb can be observed. At other seasons of the year, the north-limb-illuminations and the south-limb-illuminations are mingled in a singular way. I propose now to trace the general effect of the uncertainty in question, as limited in its application by the peculiarity of observation which I have described.

Suppose that a semidiametral correction larger than that which is just is applied to all observations of North Polar Distance. When the north limb is observed, this will increase the inferred North Polar Distance of the centre, and this applies to the summer observations. When the south limb is observed, the supposed too-large correction will diminish the inferred North Polar Distance of the centre; and this applies to the winter observations. Therefore the plane of the Moon's orbit is made to approach too near to the plane of the terrestrial equator. When the Moon's ascending node is at the first point of *Aries*, the inclination of her orbit to the ecliptic is thus diminished; and when the ascending node is at the first point of *Libra*, this inclination to the ecliptic is increased. Thus there is made to appear, in the value of the Inclination of the Moon's Orbit, a periodical change whose period is 19·2 years (the same as the period of nutation). The same result, but with opposite sign, occurs if the employed semidiametral correction is too small.

I have made great efforts to reduce the extent of this uncertainty. Since I have had the command of an Observatory, I have always carefully prepared the plan of observations of each following year before the end of the preceding year. In doing this at the Royal Observatory, a calculation is made for every day on which the Moon can be observed on the meridian, showing the state of the Moon's illumination; and from this, the possibility of using double observations of the limbs for the correction of semidiameters is immediately seen. The number available in Right Ascension is very small; the number of those which can be used in North Polar Distance is larger. I have laid down the rule, that the observations to be employed shall only be those of the gibbous Moon, and that the defect of illumination upon the imperfect limb of the Moon shall not exceed three degrees of the Moon's surface. The results of these are given in the annual volumes of *Greenwich Observations*, and it will be seen there that a considerable amount of information on this important element has been collected.

In my discussion of the Corrections to the Elements of the Moon's Orbit, as inferred from the observations made at Greenwich from 1750 to 1830, printed in Volume XVII. of the Society's *Memoirs*, I have found on page 54, as one of the results obtained

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by examination of the observations of N.P.D., an inequality in the inclination expressed by

$$-2''\cdot02 \times \text{sine longitude of node.}$$

And in the extension of this discussion, including the results of observations from 1750 to 1851, printed in Volume XXIX. of the *Memoirs*, I have found on page 21,

$$-1''\cdot87 \times \text{sine longitude of node.}$$

It is very probable that these terms may have originated, at least in part, in erroneous value of the semidiameter employed in the reductions.

It is proper to remark that any important error in refraction, applying with different magnitudes to the summer observations and the winter observations, will mix its effects with those of erroneous semidiameter.

Royal Observatory, Greenwich,
1871, *November* 15.

On the Geodesic Lines on an Ellipsoid. By Prof. Cayley. (Abstract.)

The fundamental equations in regard to the geodesic lines on an ellipsoid were established by Jacobi, viz., representing by a, b, c, the squares of the semiaxes, that is, taking the ellipsoid to be

$$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 1,$$

(where $a > b > c$), if we introduce the elliptic co-ordinates h, k, and write

$$\frac{x^2}{a+h} + \frac{y^2}{b+h} + \frac{z^2}{c+h} = 1,$$

$$\frac{x^2}{a+k} + \frac{y^2}{b+k} + \frac{z^2}{c+k} = 1,$$

or, what is the same thing,

$$x^2 = \frac{a\,(a+h)(a+k)}{(a-b)(a-c)},$$

$$y^2 = \frac{b\,(b+h)(b+k)}{b-c)(b-a)},$$

$$z^2 = \frac{c\,(c+h)(c+k)}{(c-a)(c-b)},$$